Using Derivative Regularization to Solve Inverse Heat Conduction Problems

Robert L. McMasters
Virginia Military Institute
McMastersRL@vmi.edu

Introduction

A common approach in dealing with inverse heat conduction problems is to use regularization in an attempt to smooth the estimated heat flux as a function of time. If the unknown heat flux consists of a series of discrete pulses, however, smoothing is undesirable. In dealing with cases such as these, any reduction in the ill-posed composition of the problem is extremely helpful.

Contrast with Previous Research

In an attempt to reduce the extent to which problems are ill-posed, various methods have been used which impart structure to the square sensitivity matrix ($\mathbf{X}^T\mathbf{X}$) by adding a regularization term, such as with Tikhonov regularization [2]. A method similar to this is presented in Reference [3] except that no prior information is required. The regularizing parameter is a scalar chosen by the user and the method typically involves minimizing the difference between the parameters being estimated and some other chosen quantities.

The present research, by contrast, utilizing the method of derivative regularization, employs the principle of matrix pre-multiplication to reduce the ill-conditioned nature of the matrix structure [4]. The use of a pre-conditioning method is fairly straightforward, however the difficult and most critical aspect of this method is the development of the pre-conditioning matrices for the specific type of problem.

As the name implies, derivative regularization employs the time derivatives of the sensitivity coefficients, and of the measured data, in developing a pre-conditioning matrix. One way of quantifying the degree to which an inverse problem is ill-posed is by evaluating the condition number of the matrix used in the final calculation of the unknown heat pulse magnitudes. The gain in ability to compute parameters resulting from the use of this method was shown in quantitative form in Reference [5] by comparing the condition number of the $\mathbf{X}^T\mathbf{X}$ matrix, as given by [6] for a non-linear parameter estimation problem.

Summary of Findings

The present research applies the derivative regularization method to the inverse heat conduction problem. Since the inverse heat conduction problem is linear, as opposed to the nonlinear problem studied in Reference [5], an investigation can be made into the error induced by varying the weighting factors applied to the first and second derivative contributions to the regularization. Figure 1 shows the total of the absolute values of the errors between the actual and the estimated heat pulse magnitudes used in a number of test cases. As can be seen in this figure, there is an optimum combination of weighting factors which can be utilized.

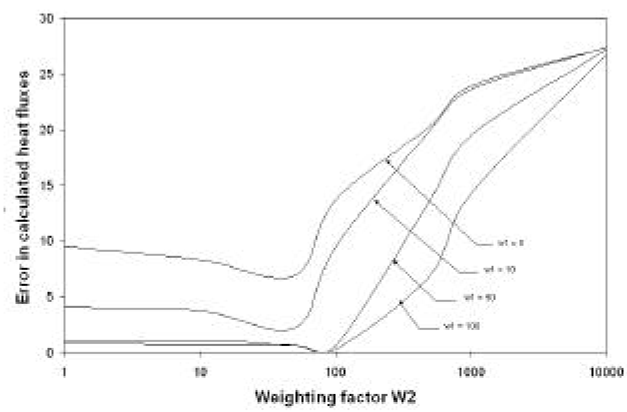

Figure 1. Error exhibited in a series of test cases using varying weighting factors on the first and second derivative components of the regularization terms. An error magnitude of 9.49 was obtained for the non-regularized case for this example.

References

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